What is the “Real” Young’s Modulus of Steel?

• R. H. Wagoner\textsuperscript{1,2}, Z. Chen\textsuperscript{2}
  • \textsuperscript{1}R. Wagoner, LLC
  • \textsuperscript{2}The Ohio State University
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Outline

- INTRODUCTION
- MATERIALS
- ULTRASONIC MODULUS ANALYSIS
- LINEAR MODULUS ANALYSIS
- NONLINEAR MODULUS ANALYSIS
- CONCLUSIONS
INTRODUCTION
The “Modulus Effect” Literature

Loading/Unloading Behavior

QPE Model vs. Experiment (Multi-Cycle)

**QPE: Performance in Springback Prediction**

Draw-bend springback measurement and prediction, DP 980. In degrees.

<table>
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<tr>
<th>F_b</th>
<th>0.3</th>
<th>0.6</th>
<th>0.8</th>
<th>0.9</th>
<th>&lt;σ°&gt;</th>
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<td></td>
<td>Δθ</td>
<td>Δθ-Δθ_{Exp}</td>
<td>Δθ</td>
<td>Δθ-Δθ_{Exp}</td>
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<tr>
<td>Experiment</td>
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<td>53.9</td>
<td>45.9</td>
<td>37.5</td>
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<td>QPE</td>
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<td>54.1</td>
<td>0.76</td>
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<td>Chaboche, Chord</td>
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<td>10.6</td>
<td>56.1</td>
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<td>Chaboche, E=208GPa</td>
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<td>-9.83</td>
<td>43.9</td>
<td>-10</td>
<td>36.5</td>
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<td>Isotropic Hard., Chord</td>
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<td>26.5</td>
<td>71.8</td>
<td>17.9</td>
<td>58.4</td>
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→ QPE reduces error of springback prediction by 50% - 65%.

<table>
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<th>Material</th>
<th>Thickness</th>
<th>Yield Stress **</th>
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<tr>
<td>Up</td>
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<td>324 MPa</td>
</tr>
<tr>
<td>lwr</td>
<td>0.8 mm</td>
<td>324 MPa</td>
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Crash Simulation: Toyota Flat Pillar

Data courtesy of U. Gandhi: *Investigation of non-linearity and anisotropy in elastic modulus of steel, TRINA, TTC, March 19, 2013*
MATERIALS
Materials (4 Grades, 3 Sources Each, RD)

- **DP 980, $\langle \sigma \rangle = 7$ MPa**
- **DP 600, $\langle \sigma \rangle = 3$ MPa**
- **HSLA, $\langle \sigma \rangle = 7$ MPa**
- **IF, $\langle \sigma \rangle = 2$ MPa**

**Tensile tests**

**Axes:**
- **Y-axis:** Engineering Stress (MPa)
- **X-axis:** Engineering Strain

- Graphs for different materials and sources are marked with numbers 1, 2, and 3.
**Definition of Hysteresis Measure, $\delta$**

Typical example: DP980-1, RD
Prestrain=0.08

- Maximum load width = $\delta_L$
  (example at 54% of $\sigma_u$)
- Maximum unload width = $\delta_U$
  (example at 47% of $\sigma_u$)

$\delta = \delta_U + \delta_L$
Is $\delta$ a Fundamental Material Property?

![Graph showing loop width ($\delta$) vs. unloading stress (MPa) for different types of steels: DP980, DP600, HSLA, and IF.](image-url)
Summary, Hysteresis Measure, $\delta$

- $\delta$ varies directly with material strength
- $\delta$ is ~independent of supplier
- $\delta$ implies a common mechanism
ULTRASONIC MODULUS ANALYSIS
Ultrasonic Procedures (96 tests)

Olympus 38DL Plus

Settings: M208-RM transducer (L)

Olympus 38DL Plus

Settings: V222-BB-RM transducer (S)
Summary, “Real” (Ultrasonic) Modulus Analysis

- Through-thickness Young’s Moduli ($E_{US}$):
  - All: 212 GPa +/- 7 GPa
  - Non-IF: 215 GPa +/- 2 GPa
  - IF: 201 GPa +/- 2 GPa

- In-plane variation ($\Delta G$): -3 GPa +/- 1 GPa (4% of $G_{av}$)

- $E_{US}$ varies little among materials, directions (<5%).
LINEAR MODULUS ANALYSIS
Mechanical Testing Procedures (192 tests)

Tensile Test Types:
- Standard
- L/U/L Single-cycle at $e_{pre} = e_u / 2$
- L/U/L Single-cycle at $e_{pre} = e_u$
- L/U/L Multi-cycle at $e_{pre} = 0.02, 0.04, 0.06, 0.08, 0.10$

Class A extensometer:
Epsilon 3542-0200-030-ST, 2” gage, +30%/-10% range

Parameters: $V = 0.08\text{mm/s}$, $\dot{e} = 0.001/\text{s}$, Data rate = 10Hz “plastic,” 1 Hz “elastic”

Parallel tensile specimen: dimensions in inches.

Single-cycle load/unload test.
Four Kinds of Mechanical Tests (192 Total)

- **Standard Tensile Test**
  - DP 600-1, RD, t=1.35 mm
  - Tensile test
  - Strain rate= 0.001/s

- **Single Cycle LUL**
  - $\varepsilon_{\text{pre}} = \varepsilon_u / 2$

- **Single Cycle LUL**
  - $\varepsilon_{\text{pre}} = \varepsilon_u$

- **Single Cycle LUL**
  - $\varepsilon_{\text{pre}} = 2, 4, 6, 8, 10\%$
Linear Modulus Analysis

![Graph showing True Stress vs. True Strain with annotations indicating points E₁, E₂, E₃, and E₄, as well as segments 1/4 σᵤ and 1/3 σᵤ.](https://www.autosteel.org)
Young’s Modulus vs. Prestrain (RD): DP980-1

Note: $E_{l} = \text{Extreme moduli from nonlinear 4-parameter fit.}$
Young’s Modulus vs. Prestrain(RD): DP600-1

DP600-1
t=1.35 mm, RD

$E_{us} = 215$ GPa

Note: $E_t$ = Extreme moduli from nonlinear 4-parameter fit.
Young’s Modulus vs. Prestrain (RD): HSLA-1

Note: $E_t$ = Extreme moduli from nonlinear 4-parameter fit.
Young’s Modulus vs. Prestrain (RD): IF-1

Note: $E_i$ = Extreme moduli from nonlinear 4-parameter fit.
Brief Statistics, Linear Modulus Analysis

All Materials
Initial Loading Modulus (8): 193 ± 6 GPa (test-test: ± 8)
Reloading (Prestrain=8%) (2): 195 ± 5 GPa (test-test ± 2)

Non-IF
Initial Loading Modulus (8): 197 ± 3 GPa (test-test ± 5)
Reloading (Prestrain=8%) (2): 191 ± 5 GPa (test-test ± 2)

Therefore:
Prestrain has little effect on E.
Use reloading instead of initial loading, better precision and reproducibility (SD 4 GPa vs. 16 GPa).
Summary, Linear Modulus Analysis

Simplifications:
• Initial E’s ~ independent of grade, supplier
• Final E’s depend primarily on $\sigma_u$
• Single-cycle = multi-cycle, $E_{RD} \approx E_{TD}$
• Small effect of prestrain on E (Contrary to Morestin)

Conclusions:
• Use unloading or reloading leg for measurement
• Initial E’s lower than $E_{us}$ (Contrary to QPE)
• Very nonlinear ($E_2, E_4$ 40~90 GPa lower than $E_1, E_3$)
NONLINEAR MODULUS ANALYSIS
Nonlinear Analysis: Data
Data: Loading vs. Unloading

DP980-1, RD
Prestrain=8%
Unload vs. Reload

Test-test scatter
(average, all materials):
Unloading: 5 MPa
QPE Model: E Form, 3-4 Parameter Versions

Sun and Wagoner’s* Eq. 11 (*), where $d\varepsilon=0$ is the linear-nonlinear transition:

$$E = A_1 - A_2 \left[ 1 - \exp \left( -A_3 |d\varepsilon| \right) \right]$$

rewritten from the strain reversal point, where $\Delta\varepsilon=0$, Eq. 1:

$$E(|d\varepsilon|) = \begin{cases} 
A_1 & \text{for } \Delta\varepsilon \leq A_4 \\
A_1 - A_2 \{ 1 - \exp \left[ -A_3 (|\Delta\varepsilon| - A_4) \right] \} & \text{for } \Delta\varepsilon \geq A_4 
\end{cases}$$

and a fully nonlinear representation ($\sigma_y=0$, $A_4=0$), Eq. 2:

$$E(|d\varepsilon|) = A_1 - A_2 \left[ 1 - \exp \left( A_3 |\Delta\varepsilon| \right) \right] \quad \text{i.e. Eq.1 with } A_4 = 0$$
QPE Model: Stress Form

The general integration form of Eq. 11 is:

\[ \Delta \sigma = (A_1 - A_2) |d\varepsilon| - \frac{A_2}{A_3} \exp[-(A_3 |d\varepsilon|)] + C \]

and the explicit form, Eq. 3, in terms of \( \Delta \varepsilon \), is as follows:

\[ \Delta \sigma = \begin{cases} 
A_1 |\Delta \varepsilon| & |\Delta \varepsilon| \leq A_4 \\
(A_1 - A_2) |\Delta \varepsilon| + \frac{A_2}{A_3} \{1 - \exp[-A_3(\Delta \varepsilon - A_4)]\} + A_2 A_4 & |\Delta \varepsilon| \geq A_4 
\end{cases} \]

and fully nonlinear representation (no yield stress), Eq. 4, is as follows:

\[ \Delta \sigma = (A_1 - A_2) |\Delta \varepsilon| + \frac{A_2}{A_3} \{1 - \exp[-A_3(\Delta \varepsilon)]\} \]
Nonlinear Fits and Data Example: DP980-1

DP980-1, RD
Prestrain=8%
All fits and data

Delta stress fraction

Delta strain fraction

Reload, 4 Param.
< S.D. >= 6 MPa

Reload, 3 Param.
< S.D. >= 9 MPa

Data, Reload
< S.D. >= 9 MPa

Unload, 3, 4 Parameters
(Indistinguishable)
< S.D. >= 4 MPa

Unload Data
< S.D. >= 4 MPa

Average statistics (all mats.):
4 Parameter Fit: 3 MPa
3 Parameter Fit: 4 MPa
Test-test Scatter: 5 MPa
Nonlinear Fit Example: Unloading

DP980-1, RD
Prestrain=8%
Unloading

Unload Data

Unload, 3, 4 Parameters (Indistinguishable)

Fit 1: \((A_1 \sim A_3)\)
Fit 2: \((A_1 \sim A_4)\)
Nonlinear Fit Example: Reloading

DP980-1, RD
Prestrain=8%
Reloading

Reload Data
Reload, 4 Parameters

Dashed line: Fit 1 ($A_1$-$A_3$)

Solid line: Fit 2 ($A_1$-$A_4$)
E vs. $\Delta \varepsilon$: DP980-1 (Unload)

- $E_{us} = 214$ GPa
- $E_{3avg} = 192$ GPa
- $E_{Chord} = 144$ GPa

Young's Modulus (GPa) vs. Delta Strain

- Fully Nonlinear Fit, A1-A3
- Linear/Nonlinear Fit, A1-A4

DP980-1, RD
Prestrain=8%
Unloading
E vs. Δε: DP980-1 (Reload)

Young's Modulus (GPa)

- Fully Nonlinear Fit, ΔA
- Linear/Nonlinear Fit, A1-A4
- E_{us}=214 GPa
- E_{avg}=192 GPa
- E_{Chord}=144 GPa

DP980-1, RD
Prestrain=8%
Reloading

Delta Strain

0 0.001 0.002 0.003 0.004 0.005 0.006 0.007

60 100 140 180 220
Summary, Nonlinear Modulus Analysis

- Equations fit within test-test scatter.

- Yielding begins at zero strain. No linear region.
CONCLUSIONS
Comparison of Modulus Measures

Non-IF Steels

Ultrasonic: 215 +/- 2 GPa
Mechanical/Linear ($\varepsilon=0$): 197 +/- 6 GPa (92% +/- 2%)
                                 (191 GPa at $\varepsilon=0.08$)
Mechanical/Nonlin. ($\varepsilon=0.08$): 215 +/- 15 GPa (100% +/- 7%)

IF Steels

Ultrasonic: 201 +/- 3 GPa
Mechanical/Linear ($\varepsilon=0$): 180 +/- 19 GPa (90% +/- 10%)
Mechanical/Nonl. ($\varepsilon=0.08$): 221 +/- 15 GPa (110% +/- 7%)
Crash Simulation: Toyota Flat Pillar

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Crash Simulation: Toyota Flat Pillar

Overall Conclusions

• No pure elasticity! Always elastic + plastic. (Simple!)

• Universal nonlinear behavior; depends mainly on strength.

• Recommend: Use unloading or reloading for measurement.

• Small differences in each material class.
EXTRA SLIDES
### Comparison of Initial Modulus Measures

#### Three Overall Modulus Measures

<table>
<thead>
<tr>
<th>Material</th>
<th>Ultrasonic (Elastic)</th>
<th>Mechanical, Linear Fit</th>
<th>Mechanical, Nonlinear Fit, Eq.4</th>
<th>Comparisons</th>
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<tr>
<td></td>
<td>$E_{us}$</td>
<td>&lt;S.D.&gt;</td>
<td>$E_3^*(e=0)$</td>
<td>&lt;S.D.&gt;</td>
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<tr>
<td>DP980-1</td>
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<td>192</td>
<td>9</td>
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<td><strong>Average</strong></td>
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<td><strong>1</strong></td>
<td><strong>193</strong></td>
<td><strong>16</strong></td>
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<tr>
<td><strong>S.D.</strong></td>
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<td><strong>1</strong></td>
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<td>DP980 Avg.</td>
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<td>IF Avg.</td>
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<td>19</td>
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*Average of $E_3$ at 0 prestrain, all types of load/unload test (8)